PCA dual problem (cont.)

And using the orthogonality of the columns of $\tilde{X}$ and G

$$\breve{B}^t \hat{\Sigma} G = \breve{B}^t \left(\tilde{X} \tilde{X}^t \right) G = \left(\breve{B}^t \tilde{X} \right) \left(\tilde{X}^t G \right) = \left(\breve{B}^t \tilde{X} \right) 0 = 0_{n \times (d-n)}$$

$$G^t \hat{\Sigma} \breve{B} = G^t \left(\tilde{X} \tilde{X}^t \right) \breve{B} = \left(G^t \tilde{X} \right) \left(\tilde{X}^t \breve{B} \right) = 0 \left(\tilde{X}^t \breve{B} \right) = 0_{(d-n) \times n}$$

$$G^t \hat{\Sigma} G = G^t \left(\tilde{X} \tilde{X}^t \right) G = \left(G^t \tilde{X} \right) \left(\tilde{X}^t G \right) = 0 \cdot 0 = 0_{(d-n) \times (d-n)}$$

PCA dual problem (cont.)

Thus:

$$B^t\hat{\Sigma}B = \begin{pmatrix} D^* & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 & & \cdots & & 0 \\ 0 & \ddots & & & & \\ & & \lambda_n & \ddots & & \vdots \\ \vdots & & \ddots & 0 & & \\ & & & & \ddots & 0 \\ 0 & & \cdots & & 0 & 0 \end{pmatrix} = D$$

PCA dual problem (cont.)

Aside about orthogonal component G:

Usually don't need to compute,

Since only care about "eigenvectors for non-zero eigenvalues"

[go to next pages]