QUEUEING ANALYSIS OF NETWORK TRAFFIC

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BASIC QUEUEING SYSTEM

- $r(t)$ = instantaneous traffic input rate at time t. (Mbps)
- Incoming traffic resides in a buffer of infinite capacity.
- c = maximum rate at which traffic can be removed from the buffer. (Bandwidth of the output channel.) (Mbps)
- $X(t)$ = amount of data in the buffer at time t.
- System Dynamics:

$$\frac{dX(t)}{dt} = \begin{cases} r(t) - c & \text{if } X(t) > 0, \\ (r(t) - c)^+ & \text{if } X(t) = 0. \end{cases}$$

BASIC QUEUEING EQUATION

- Total data input upto t

$$A(t) = \int_0^t r(s)ds.$$

- Total output upto time t

$$\begin{aligned} D(t) &= \int_0^t c1_{\{X(s)>0\}}ds + \int_0^t r(s)1_{\{X(s)=0\}}ds \\ &= ct + \int_0^t (r(s)-c)1_{\{X(s)=0\}}ds. \end{aligned}$$

- $Y(t) = A(t) - ct =$ netput process.
- Conservation of Traffic Equation:

$$\begin{aligned} X(t) &= X(0) + A(t) - D(t) \\ &= X(0) + \int_0^t r(s)ds - ct - \int_0^t (r(s)-c)1_{\{X(s)=0\}}ds \\ &= X(0) + Y(t) - \int_0^t 1_{\{X(s)=0\}}dY(s) \end{aligned}$$

SAMPLE PATH SOLUTION

- The basic queueing equation has the following sample path solution:

$$
\begin{aligned}
X(t) &= X(0) + Y(t) - \min\{0, \inf_{0 \le u \le t} (X(0) + Y(u))\} \\
&= \max\{X(0) + Y(t), \sup_{0 \le u \le t} (Y(t) - Y(u))\}
\end{aligned}
$$

- See picture.

STABILITY AND LIMITING SOLUTION

- Condition of stability:

$$\lim A(t)/t < c.$$

- Suppose $\{r(t), t \geq 0\}$ is stationary with $E[r(t)] < c$. Then

$$X^* = \sup_{u \leq 0} \int_u^0 (r(s) - c)ds$$

is an a.s. finite random variable, and

$$\lim_{t \to \infty} P(X(t) > x) = P(X^* > x).$$

EXAMPLE 1: ON-OFF SOURCE

- Let $\{Z(t), t \geq 0\}$ be a CTMC on $\{0, 1\}$ with generator matrix:

$$Q = \begin{bmatrix} -\alpha & \alpha \\ \beta & -\beta \end{bmatrix}.$$

  The CTMC stays in state 0 for an $\exp(\alpha)$ amount of time and then moves to state 1, stays there for an $\exp(\beta)$ amount of time and then moves to state 0, and continues this way forever.

- $r(t) = rZ(t)$. Such a source is called a Markovian on-off source. It generates traffic at rate r for an $\exp(\alpha)$ amount of time (on-time), then generates no traffic for an $\exp(\beta)$ amount of time (off-time) and oscillates this way forever.
- Long run mean input rate: $m = r\frac{\beta}{\alpha+\beta}$.
- Condition of stability: $m < c$.
- When the queue is stable:

$$P(X > x) = \frac{m}{c} e^{-\lambda x},$$

  where

$$\lambda = \frac{\alpha}{r - c} - \frac{\beta}{c} > 0.$$

EXAMPLE 2: GENERAL MARKOVIAN SOURCE

- Let $\{Z(t), t \geq 0\}$ be an irreducible CTMC on finite state space S with generator matrix Q and stationary distribution π.
- $r(i)$ = input rate in state i.
- Mean input rate = $m = \sum_{i \in S} \pi(i) r(i)$.
- Condition of stability: $m < c$.
- D = a diagonal matrix with $D_{i,i} = r(i) - c$.
- For a stable system:

$$P(X > x) \sim e^{-\lambda x},$$

where λ is the smallest positive solution to

$$det(\lambda D + Q) = 0.$$

- In fact:

$$P(X > x) \leq e^{-\lambda x}.$$

EXAMPLE 3: MARKOV MODULATED BROWNIAN SOURCE

- Let Z be as before.
- $r = [r(i)]$, $\sigma = [\sigma(i)]$.
- $\{W(t), t \geq 0\}$: standard white noise. $W(t)$ is a $N(0,1)$ random variable.
- $r(t) = r(Z(t)) + \sigma(Z(t))W(t)$.
  Such a source is called a Markov modulated Brownian source. The instantaneous input rate in state i is a $N(r(i), \sigma^2(i))$ random variable.
- Stability condition same as before.
- For a stable system:

$$P(X > x) \sim e^{-\lambda x},$$

  where λ is the smallest positive solution to

$$det(\frac{1}{2}\lambda^2\Sigma^2 + \lambda D + Q) = 0.$$

- In fact:

$$P(X > x) \leq e^{-\lambda x}.$$

SOME OBSERVATIONS

- All these models produce exponential tails for the buffer content in steady state.
- At variance with observed data. Hence we need some other models of traffic sources.
- Common feature: Short range dependence of the $\{r(t), t \geq 0\}$ process.
- We need to consider long range dependent processes if we want to get away from the exponential tails.
- Candidates: Fractional Gaussian processes, self-similar processes, Conservative Cascades. The first two are stationary, the last is not.